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Model-Based Monitoring of Diamond Turning Process Using Cross Entropy

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This paper discusses the model-based monitoring as one of key steps toward building an in-process diagnostic tool for the supervisory level control in diamond turning. In particular, estimation of surface profile from a nonlinear force model and model-based monitoring of tool chipping are demonstrated. Classification rule is based on the minimization of information-theoretic measure called the cross entropy. The experimental results indicate that monitoring strategy based on the minimum cross entropy guarantees the consistent and optimum classification results in the presence of varying cutting conditions. In addition, the cross entropy as a criterion for indicating the dynamic quality of machine-tool is explained.

Key Words: Diamond Turning, Monitoring, Cross Entropy, Surface Profile, Chipping

Nomenclature

a_i, σ : Parameters of AR process
a, K, μ : Constants
f, f_{crit} : Feed rate, Critical feed rate
f_{Nyq} : Nyquist frequency
f_s, f_T : Spatial frequency, Sampling frequency
$F_y(T)$: Instantaneous cutting force component
F_0 : Nominal value of
ΔF_0 : Instantaneous change in
$H(t_1)$: Hardness of diamond turned surface
$H[q, q_i]$: Cross entropy calculated from the spectral densities
i : Index for the local nodes
j : Index for the global nodes in the surface profile
M : Number of segments on the cutting edge that are spaced with equal length Δ_x in the feed direction
n : Spindle speed
N : Number of data points (global nodes) in the surface profile
N_r : Number of revolutions
p, q : Probability densities
R : Tool nose radius
T : Time
ΔT : Sampling period
$t_1(T)$: Instantaneous depth of cut
t_0 : Nominal value of
$\Delta t_1(T)$: Instantaneous change in $t_1(T)$
(x^i, y^{ir}) : Position of the i_{th} th node on the cutting edge in the r_{th} revolution
(X^{or}, Y^{or}) : Position of tool relative to the workpiece in the r_{th} revolution
y_p^j, y_q^j : Surface profiles associated with the densities p and q, respectively
y_f^j : Surface profile due to feed mark
y^{jr} : Position of the cutting edge at j in the r_{th} revolution

1. Introduction

The implementation of an intelligent controller in machining operations often requires the integration of a wide array of sensors, signal processing technology, and decision making algorithms to assess the current state of cutting process and machine-tool. The intelligent controller, in most cases, should work to synthesize the observational data from the sensors and the heuristic knowledge, and to support the engineering decision making. Ulsoy and Koren (1993) proposed a

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classification for the different levels of intelligent control in machining operations. The lowest level is servo control in which the motion of cutting tool relative to the workpiece is controlled. The next level is process control where process variables such as cutting forces, cutting power and cutting temperature are controlled to maintain high productivity and part quality. The highest level is supervisory control in which the control strategies are aimed at compensating for factors not explicitly considered in the design of servo and process level controllers. The emphasis at this level is placed on the capability for integrated monitoring and control in real-time. Examples in monitoring applications include tool monitoring, machine monitoring, chatter detection, etc.

A framework that integrates the differential functions of a turning system is shown. The system consists of two main components: surface generation process and an intelligent controller. The cutting process coupled with the machine-tool dynamics and the associated regenerative effect is termed as the surface generation process since only the generated surface profile is considered as an output of the system in this paper. The inputs to the surface generation process consist of cutting conditions and tool material/geometry, and the disturbance inputs may include cutting tool wear/breakage, chip breakage, chatter, etc.

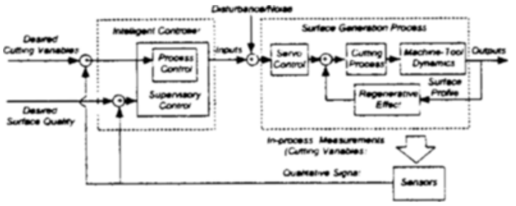

Fig. 1 A framework that integrates differential functions of a turning system.

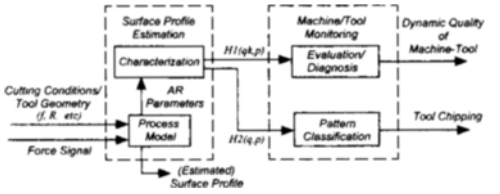

Fig. 2 Structure of a proposed monitoring system.

The purpose of this study is to propose a model-based monitoring method as one of key steps toward building an in-process diagnostic tool for the supervisory level control in diamond turning. In this study, the monitoring strategy is divided in two steps, modeling step, and charcterization and pattern classification step. In the first step, a theoretical model for predicting the surface profile in the presence of random cutting tool vibration in diamond turning is presented. Autoregressive modeling of generated surface profile is performed next, followed by the characterization of surface profile using the concept of cross entropy. Pattern classification is based on the minimization of this cross entropy. Then, the experimental relationship between these surface characteristics and the tool chipping is established. The robustness of established relationship under varying conditions is also addressed. In addition, the cross entropy as a criterion for indicating the dynamic quality of machine-tool is explained. The overall structure of a proposed monitoring system is also illustrated.

2. Monitoring of Tool Chipping in Diamond Turning

2.1 Chipping of diamond tool

The wear of a diamond tool can be conveniently classified into two types, one of which is chipping and the other is gradual wear (Ikawa, Shimada and Morooka, 1987). Diamond is a brittle material. Therefore, fine cutting edge may degenerate by tool chipping which arises from natural imperfection or careless handling. In diamond turning, tool chipping needs to be avoided since it may lead to catastrophic fracture even before the expected tool life. In addition, it degrades the surface quality severely. Since the surface profile is the outcome of machining process and machine-tool interaction there must exist a correlation between the surface profile and the characteristics of the cutting process, one of which would be the state of tool chipping. Any process model describing surface generation process must incorporate the effects of machine-tool dynamics as well as cutting process on the surface into its formulation. Then, by linking cutting process and surface generation model one can predict the surface profile of a machined part, thereby allowing the prediction of tool chipping.

2.2 Model-based monitoring

Successful implementation of a monitoring and control strategy at the supervisory level typically requires a realistic process model based on the physical understanding of the process. One of the advantages in model-based approach is that rules and procedures required for rendering a diagnosis based on sensor outputs are simplified (Stein, 1993). In addition, improvement of sensor performance can be made by model-based estimation in which an estimation algorithm is used to extract the relationship between the sensor signal and the quantity to be measured. The process model can be either a mechanistic one or an empirical one.

3. Modeling of Surface Profile Generation

Several advantages of using a diamond tool in machining operations have previously been recognized (Wilks, 1980). However, from the point of view of surface generation process, the most noteworthy advantage is the low friction at the tool-chip and the tool-workpiece interfaces, which makes the chip formation under eased stress condition possible. In addition, the extreme hardness of diamond permits the fabrication of very sharp cutting edge which will produce very fine surface. The small scaled surface roughness stemming from the physical factors may be neglected for reasons stated and any randomness in the profile is mainly due to tool vibration in diamond turning.

3.1 Modeling of cutting tool vibration

In order to model the surface profile generation, one needs to estimate the cutting tool vibration first. We consider a case that a cylindrical workpiece is diamond turned without any influence of physical factors. Machine-tool dynamics is assumed to be a single degree of freedom system in the workpiece radial direction (y direction in Fig. 3). Other assumptions are summarized in Appendix. Among input variables in actual turning operations such as cutting speed, depth of cut, feed rate and tool geometry (tool nose radius R), the interaction between feed rate and tool geometry was recognized as a dominant factor in defining the surface profile (Rakhit, Sankar and Osman, 1976). The process dynamics at the micro chip formation level and the macro machine-tool level may then be specified by feed rate and tool geometry.

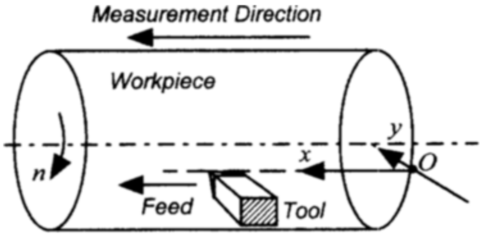

Fig. 3 Measurement of surface profile formed in turning operations.

Typically, the complexity of cutting process dynamics stems from the nonlinear behavior of cutting process and machine-tool structure. According to the force model developed by Drescher, and Dow(1990), the radial force in a single point diamond turning is given by:

$$\begin{aligned} F_y(T) &= F_0 + \Delta F_y(T) \\ &= \mu\left(H_0 f t_1(T) + \frac{RK}{\alpha}\theta_{max} \right. \\ &\qquad \left. + \frac{RK}{\alpha^2 f}\left(e^{-\alpha f \theta_{max}} - 1\right)\right) \end{aligned} \tag{1}$$

where $\cos\theta_{max} = 1 - t_1(T)/R$. An exponentially increasing hardness near the diamond turned surface was assumed in Eq. (1), i. e.,

$$H(t_1) = H_0 + Ke^{-\alpha t_1(T)} \tag{2}$$

H_0 and K in Eq. (2) for Al6061-T6 are 40 kg/mm^2 and 130.5kg/mm^2, respectively. The constant α is 3.26. The constant μ depends on the friction at the tool-chip interface and has a value close to 1.0 for Al 6061-T6. We now represent the instantaneous depth of cut $t_1(T)$ as the sum of t_0 and $\Delta t_1(T)$, i. e.,

$$t_1(T) = t_0 + \Delta t_1(T) \tag{3}$$

$\Delta t_1(T)$ equals the cutting tool vibration in time, (X^{or}, Y^{or}) and can be estimated from $\Delta F_y(T)$ for given nominal depth of cut, feed rate and tool geometry by solving Eq. (1).

3.2 Estimation of surface profile

Since the most important geometric information on turned surface is contained in the tool feed direction, measurements (or estimations) are made in this direction with the sampling period ΔT that is equal to the interval for a single feed (one data point per revolution). An analytical model describing surface profile generation in diamond turning was developed by Choi (1994). In actual turning operations, tool may recut the surface formed in previous revolutions. In particular, cutting action at a particular node j in Fig. 4 occurs only if the position of cutting edge in the current rth revolution is deeper into the workpiece than that in the previous revolutions. Similarly, the profile at j may be further trimmed in revolutions to come so that total of $(2P+1)$ solutions are possible for y_q^j. However, the physically plausible solution satisfies :

$$y_q^j = y^{jl} \geq y^{jk} \text{ for } r-P \leq k \leq r+P$$
$$l \neq k, \quad j=1, \cdots, N \tag{4}$$

where j, y^{jr} and P are given in Appendix. N is the total number of data points (global nodes) in the profile concerned and the estimated surface profile y_q^j comprises $Y_q = (y_q^1, \cdots, y_q^N)$.

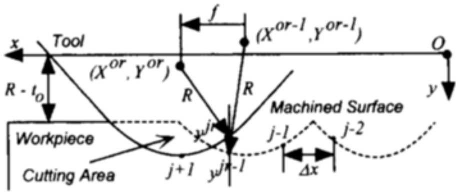

Fig. 4 Designation of the global index.

4. Monitoring Strategy

It may be assumed that the overall tool geometry is not altered significantly by chipping. As tool wears out by chipping, however, the assumption of sharp cutting edge becomes no more valid. More significantly, the contact condition at the tool flank face may change, which results in the deviation from the established relationships among the characteristics of surface profile for fresh tool.

The sensitivity of force signal to tool chipping is extremely low in diamond turning. Consequently, no noticeable difference in the average force level is observed in case of tool chipping. Frequency analysis of surface profile reveals a shift of energy to the high frequency region. However, an examination of force signal in frequency domain alone does not guarantee the robustness of established relationships to changes in cutting conditions and environment. Then, surface characteristics that are sensitive to tool chipping but insensitive to changes in cutting conditions and environment need to be identified. Otherwise adaptive method needs to be implemented, which makes practical implementation more difficult.

4.1 Characterization by cross entropy

Let y be a state of some system that has a set D of possible states. Let D be the set of all possible probability densities q on D such that $q(y \in D) \geq 0$ and

$$\int_D q(y)\,dy = 1 \tag{5}$$

The entropy of a system (or a process) with the probability density q is represented as :

$$E[q] = -\int_D q(y) \log q(y)\,dy \tag{6}$$

The entropy is a measure of the amount of information produced by a random process, or a measure of uncertainty in a random process. The larger value of entropy corresponds to more information (uncertainty) in the process. The cross entropy is a generalization of entropy when the prior density p is available, and given by:

$$H[q, p] = \int_D q(y) \log\left(\frac{q(y)}{p(y)}\right) dy \tag{7}$$

Eq. (7) states that the total amount of information produced by a process equals the sum of the amount of information gained by the posterior (current) density q and the information already acquired by p. The priors must be strictly positive, i. e.,

$$p(y \in D) > 0 \tag{8}$$

Suppose that q and q_i are referred as the spectral power probability densities of stationary random processes y_q and y_{qi}, respectively. $y_{qi} = (y_{qi}^1, \cdots, y_{qi}^N)$, $i=1, 2$, is a predefined pattern vector. One way of quantifying the distortion between the spectra of y_q and y_{qi} is to use the cross entropy defined in terms of Itakura-Saito distortion measure (Shore and Johnson, 1981). Since an autoregressive (AR) process represents the stationary time correlation of process, the surface profile can be coded in terms of AR parameters by estimating the autocorrelation function of surface profile. If the spectrum of y_{qi} has a limited bandwidth and is smooth in that band, y_{qi} can be represented by the autoregressive (AR) process of order S :

$$y_{qi}^j = -\sum_{k=1}^{S} a_k y_{qi}^{j-k} + \sigma u^j \tag{9}$$

The cross entropy functional for Itakura-Saito distortion measure is given by (Shore and Gray, 1982) :

$$H[q, q_i] = \frac{1}{\sigma^2}\left\{ r_x(0)\, r_a(0) + 2\sum_{s=1}^{S} r_x(s) \, r_a(s) \right\} + \log(\sigma^2) \tag{10}$$

where

$$r_a(s) = \begin{cases} \sum_{i=0}^{S-s} a_i a_{i+s}, & s \le S \\ 0, & \text{otherwise} \end{cases} \tag{11}$$

where $a_0=1$, σ is a filter gain and u^j is a zero mean, unit variance Gaussian sequence. $r_x(s)$'s are the autocorrelation functions of the current estimate y_q for lags $s=0, 1, \cdots, S$.

4.2 Pattern classification

The consistent and optimal classification procedure based on the minimum cross entropy becomes (Shore and Gray, 1982)

1. Compute $H[q, q_i]$ for $i=1, 2$.
2. Find i such that $H[q, q_i] \le H[q, q_j]$ for $i \ne j$.

The cross entropy based on Eqs. (10) and (11) are known to be particularly useful in real-time application due to their simplicity in computation. It has a benefit over the traditional Fast Fourier Transform (FFT) analysis.

Table 1 Cutting conditions for diamond turning experiments.

Work material	Al6061-T6 bar
Work diameter	50.8*mm* (2*in*)
Tool nose radius, R	Polycrystalline diamond (insert type)
Tool clearance angle	11°
Spindle speed, n	1200*rpm*
Nominal depth of cut, t_0	0.051~0.203*mm*
Feed rate, f	0.051~0.203*mm/rev*
Constants, α, μ	3.26, 1.0, respecitively

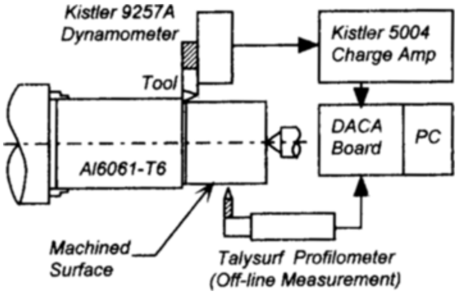

Fig. 5 Schematic of data acquisition setup.

5. Experiments

Two different series of diamond turning experiments were conducted on a Tree 1000 lathe. The first set of experiments was performed with a fresh tool (Kennametal TPG 321 diamond insert) and the second set with a chipped tool. After 20 minutes of operation, each fresh tool was examined using a microscope to check the occurrence of chipping on the cutting edge. The schematic of data acquisition setup used is shown in Fig. 5. For measurements of force signal, Kistler 9257A dynamometer was used. The tool was mounted on top of dynamometer. The measured signals were fed into Kistler 5004 charge amplifier, passed through a lowpass filter, and sampled in a PC at a rate of 1024 H_z. The true profiles of machined surface were measured using Talysurf profilometer and denoted as $y_{q+} = (y_{q+}^1, \cdots, y_{q+}^N)$.

Cutting conditions and constants used in the experiments are summarized in Table 1. Δx was set to $f_{min}/10$ so that Q has a value between 10 and 40 depending on feed rate ($f_{max}=4f_{min}$). Other parameters used were :$M=80$, $N=1000$ and $N_r=100\sim25$ depending on feed rate. For calculation of $H[q, \ q_i]$, all the sample sequences were normalized so as to have unit energy.

6. Results and Discussion

The surface profile sampled along the feed direction over the length of $1mm$ is shown in Fig. 6 for $t_0=0.102\ mm$ and $f=0.102\ mm/rev$. The figure indicates that the waviness and the form error, which contributes to the power in the low frequency region, may be neglected.

Figure 7 shows the variation of $H[q, \ q_1]$ with $H[q, \ q_2]$ calculated from Eq. (10) for $S=5$, and different combinations of t_0 and f. q_1 and q_2 were obtained from sample profiles for a fresh tool and a chipped tool, respectively. According to the classification rule, the region above the solid line represents the state of fresh tool, whereas the region below the line signifies the state of chipped tool. It is observed that two classes were reasonably separated, implying that the decision on the state of cutting tool can be made by quantitatively analyzing the surface profile generation. In doing that, changes in cutting conditions were taken into account by estimating the surface profile from the model developed. This enables the decomposition of spectral distortion induced solely by tool chipping. Consequently, through process modeling and pattern classification based on the minimum cross entropy, one can obtain not only the consistent and optimal classification results but also the robustness of established relationship for monitoring purpose.

It has been analytically shown (Moon and Sutherland, 1994) that the spatial frequencies due to feed and its harmonics will be present in the surface profile. Furthermore, wavelengths present in the surface profile can be obtained through a simple superposition of the wavelengths due to feed and those due to machine-tool dynamics. For example, Fig. 8 shows the power spectrum of surface profile calculated from the

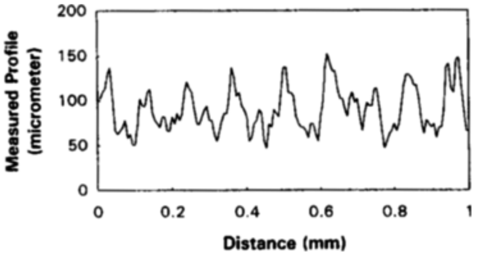

Fig. 6 Example of surface profile measured over the length of $0.1mm$ for $t_0==0.102mm$ and $f=0.102mm/rev$.

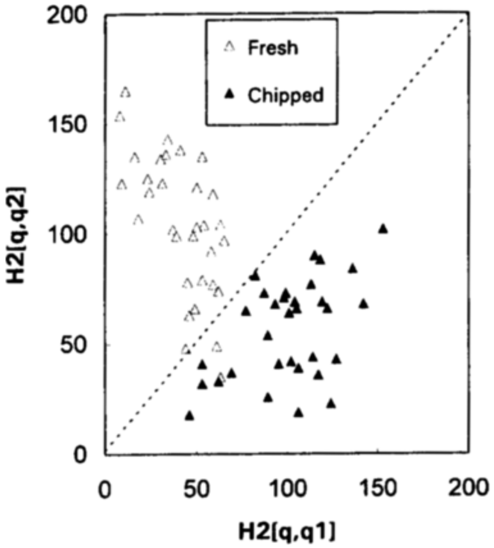

Fig. 7 Variation of $H[q, \ q_1]$ with $H[q, \ q_2]$ for fresh and chipped tools. $t_0=0.051\sim0.203mm$ and $f=0.051\sim0.203mm/rev$.

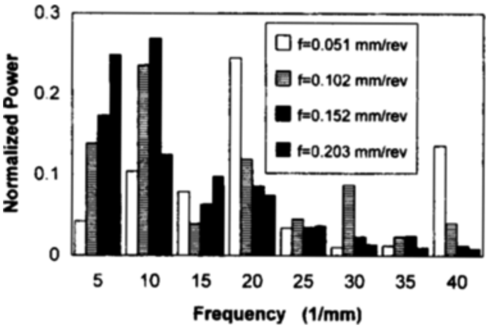

Fig. 8 Power spectrum calculated from y_q for $t_0=0.102mm$.

model developed for cutting conditions indicated. Since one revolution of workpiece corresponds to a sample interval of feed, one can define a relationship that converts the time frequency f_s :

$$f_s = \frac{f_T}{2f_{Nyq}f} \tag{12}$$

where Nyquist frequency f_{Nyq} is equal to $1/2\Delta T$. In the figure, the fundamental frequency corresponding to feed rate of $0.102 mm/rev$ is $9.84 mm^{-1}$ and the variation of profile around this frequency is due to feed mark. The higher order harmonics of feed mark contribute to the high frequency components of surface profile. In addition, the frequency characteristic of sampled signal is distorted by the aliasing. However, aliasing will add higher frequency components beyond the Nyquist frequency (10 Hz) by frequency folding. Therefore, the spectral energy below the fundamental frequency reflects the machine-tool dynamic effects.

The spectral distortion based on Itakura-Saito distortion measure in Eq. (10) weights the local maxima more heavily than the local minima (Markel and Gray, 1976). Let

$$A_s(z) = 1 + \sum_{k=1}^{s} a_k z^{-k} \tag{13}$$

in z domain where $z = \exp(2\pi i \Delta T f_T)$ and a_k is given in Eq. (9). Also, define $V(A_s)$ as :

$$V(A_s) = \log\left|\frac{y_q(e^{j\theta})}{\sigma/A_s(e^{j\theta})}\right|^2 \tag{14}$$

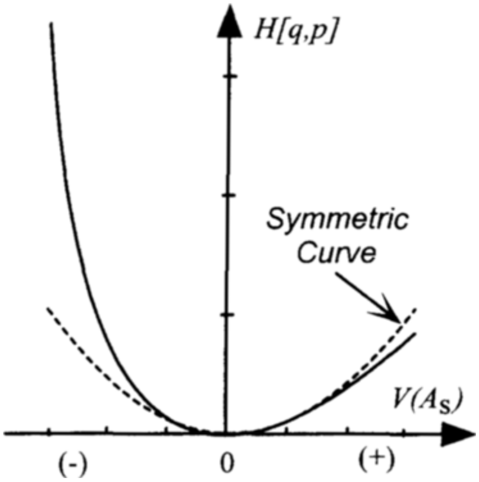

Fig. 9 Non-uniform weighting by Itakura-Saito distorion mearsure.

$V(A_s)$ is the log spectrum of inverse filter output, normalized by a filter gain σ. When $V(A_s) < 0$, relatively large distortion is generated in Eq. (10). On the contrary, small distortion is made when $V(A_s) > 0$, Fig. 9. If the energy of one spectrum was constrained to match the energy of the other by normalization as in our example, small error contribution can not be introduced by arbitrarily placing one spectrum far below the other. Therefore, the distortion measure in Eq. (10) tends to reflect more accurately the spectral peak below the fundamental frequency due to feed mark that are induced by the machine-tool dynamics.

7. Conclusion

The approach presented in this paper provides an in-process diagnostic tool for monitoring the surface generation process and the state of tool chipping. The inherent limit of this study lies in the fact that the proposed model does not accounts for the effects of physical factors on surface profile generation. However, as previously mentioned, the importance of such effects may be neglected for various reasons in diamond turning. With the model describing the interaction between the tool and the workpiece, geometric-kinematic aspects in surface generation process were taken into account in this study.

In monitoring of tool chipping, the classification rule is based on the minimization of $H[q, q_i]$ calculated from AR parameters of surface profile. Changes in cutting conditions were taken into account by model-based approach. By linking cutting conditions and surface generation process, one can predict the response in surface finish to variation of cutting conditions, which leads to a better definition of a machined surface. As a result, the robustness of monitoring strategy turned out to be preserved even in the presence of varying cutting conditions. Finally, the dynamic quality of machine-tool was evaluated using the cross entropy. The cross entropy defined in terms of Itakura-Saito distortion measure quantifies the spectral distortion and has a benefit in real-time application. Such a distortion measure based on non-uniform weighting was proved to be effective for evaluation of spectral peaks below the fundamental frequency due to feed mark that are induced by the machine-tool dynamics.

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Appendix

The following assumptions are made in modeling the surface profile generation process:

[1] Feed rate is kept sufficiently small such that feed mark is independent of nominal depth of cut, t_0, i. e., $f \ll f_{criT} = 2\sqrt{t_0(2R - t_0)}$

[2] The tool does not leave the cut and remains in contact with the workpiece, i. e., $y^{jr} \geq R - t_1(T)$ for all j

[3] Non-linearity caused by the instantaneous clearance angle is neglected by using a tool with sufficiently large clearance angle.

[4] Sampling period is equal to the interval for a single feed, i. e., $\Delta T = 60/n = 1/f_T$ (sec)

Expressions that are used in the main frame of the text are presented here for ease of reference. Interested readers should refer to (Moon and Sutherland, 1994) for more details.

$$x^{ir} = X^{or} + \Delta_x\left(i - \frac{M}{2}\right), \quad 0 \leq iM \tag{15}$$

$$y^{ir} = Y^{or} + \sqrt{R^e - (x^{ir} - X^{or})^e} \tag{16}$$

$$j = (R - 1)Q + i \tag{17}$$

$$y^{jr} = \begin{cases} 0, & r < 0 \\ y^{ir}, & 0 \leq r \leq N_r \end{cases} \tag{18}$$

where M is a even number, and integers P, N_r and satisfy :

$$P = \frac{M}{2Q} \tag{19}$$

$$N_r = \frac{N}{Q} \tag{20}$$

$$Q = \frac{f}{\Delta_x} \tag{21}$$